

Fermat's last theorem

A seventeenth century puzzle solved

Peter Symonds
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On June 23, 1993, an event took place at the Isaac Newton Institute for Mathematical Sciences at Cambridge University in Britain of considerable historic significance for the field of mathematics.

In what could prove to be a major breakthrough, Andrew Wiles, a 40-year-old number theorist from Princeton University, concluded a series of three lectures on "Modular forms, elliptic curves, and Galois representations" by proving one of the longest standing problems in mathematics--Fermat's last theorem, first stated around 1637.

According to press reports, the proof, which is yet to be published, is up to 1,000 pages in length and uses intricate arguments from highly abstract areas of pure mathematics. Wiles' claims are yet to be checked in detail and it is possible that a flaw will be found. [Note: A major gap was found in the proof and was only resolved in October 1994. After extensive checking, the manuscripts were finally published in the May 1995 volume of the journal *Annals of Mathematics*.]

In fact, for more than 350 years, Fermat's last theorem has defeated the best mathematicians of the day and is notorious for a large volume of false proofs provided by lesser amateurs and professionals.

One writer on the history of mathematics, Eric Bell, made a point of noting on Fermat's last theorem: "In passing, may I request any reader of this section who imagines he has a proof not to send it to me? I have examined well over a hundred fallacious attempts, and I feel that I have done my share. One such, many years ago, stuck me for three weeks. I felt that there was a mistake, but couldn't find it. In desperation I turned the author's manuscript over to a very bright girl in my trigonometry class, who detected the blunder in half an hour.

"Anyone contemplating a proof may be interested in what [David] Hilbert said in 1920 when asked why he did not try: 'Before beginning I should have to put in three years of intensive study, and I haven't that much time to squander on a probable failure.'"

Hilbert was the leading mathematician of the period.

According to a number of leading number theorists who attended the Cambridge lectures, the proof given by Andrew Wiles, which builds on a series of more recent developments in pure

mathematics, is convincing.

Kenneth Ribet, a mathematician from the University of California, Berkeley said: "Wiles has a first-rate reputation in the subject. He is careful, and he is methodical; he does very, very good work ... and he presented beautiful arguments."

It appears that a lengthy chapter in the history of mathematics may have finally been closed.

The theorem was asserted in 1637 by Pierre de Fermat, a French lawyer and amateur mathematician, who played a prominent role in the founding of number theory, the area of mathematics dealing with the properties and inter-relationships of whole numbers.

Both the ancient Greeks and Babylonians were aware that the equation $x^2 + y^2 = z^2$ had whole number solutions. If x , y and z are 3, 4 and 5 respectively, then $3^2 = 9$ plus $4^2 = 16$ is equal to $5^2 = 25$. Another such solution is if x , y and z are the numbers 5, 12 and 13. There are an infinite number of such whole number solutions, known as Pythagorean triples.

While studying the work of the Greek mathematician Diophantus who lived around 250 AD in Alexandria, Fermat generalised the equation to consider numbers raised to any power.

In a famous note in the margin of his copy of Diophantus' *Arithmetica*, Fermat wrote: "To resolve a cube into the sum of two cubes, a fourth power into two fourth powers, or in general any power higher than the second into two of the same kind, is impossible; of which fact I have found a remarkable proof. The margin is too small to contain it."

Put more concisely, the equation $x^n + y^n = z^n$ has whole number solutions if $n = 2$ but for all larger values of n , Fermat asserted that there were no whole number solutions.

Fermat never wrote down his "remarkable proof" and mathematicians since then have cast doubt on whether, given the mathematical techniques available in the seventeenth century, it ever existed.

It took more than 100 years before Leonhard Euler, a mathematical genius whose work encompassed the entire field of mathematics as well as physics and astronomy, proved Fermat's last theorem when $n = 3$ and $n = 4$.

Nearly another century elapsed before Peter Lejeune Dirichlet in 1828 and Adrien-Marie Legendre in 1830 proved the theorem for $n = 5$. In 1832, Dirichlet found a proof for $n = 14$.

The famous mathematician Carl Friedrich Gauss tried to solve the problem and failed. In disgust, he wrote to Heinrich Olbers: "I confess indeed that Fermat's theorem as an isolated proposition has

little interest for me, since a multitude of such propositions, which one can neither prove nor refute, can easily be formulated."

However Ernst Kummer, who studied under Gauss and Dirichlet, created an entire new theory of "ideal" numbers and the modern algebraic concept of an "ideal" in a bid to solve the problem.

As one author described Kummer's work: "Armed with this new weapon, which must be wielded with the skill of a master duelist, parrying from ordinary to ideal numbers or riposting back again when the right opening appears, Kummer proved Fermat's last theorem for exponents n that are 'regular' primes. There is a somewhat technical definition of what this means: suffice it to say that among primes less than 100 it covers all save 37, 59 and 67. Using additional arguments, Kummer and Dimitri Mirimanoff dealt with these cases too." [*The problems of mathematics*, Ian Stewart, p.29]

Mathematicians extended Kummer's techniques and by 1980 had proven Fermat's last theorem for every case up to $n = 125,000$. Of course, an infinite number of cases remained.

In 1920, British mathematician Leo Mordell came up with a general conjecture, one of the consequences of which was that for n greater than 2 there were at most a finite number of solutions to the Fermat equation. The Mordell's conjecture was finally shown to be correct in 1983 by Gerd Faltings, using powerful new methods from algebraic geometry.

Soon afterwards, D.R. Heath-Brown proved that Fermat's theorem was true for "almost all" powers but the final proof still eluded mathematicians until last month.

In examining the lengthy history of this problem, it is worth making one final point.

The proof by Wiles consists of highly abstract mathematics. As Kenneth Ribet commented: "Wiles' arguments are based on the most advanced, most elaborate mathematics that exist in this field. The number of mathematicians who can really understand the arguments would fit into a conference room."

The character of such mathematics and related areas of science has led some commentators, scientists included, to speculate that mathematics is not of the material world but of some separate and mystic nether world.

An example is contained in the book, *The Mind of God* published last year by theoretical physicist Paul Davies, who has gathered together a series of old philosophical arguments for the existence of God and dressed them up with terminology and examples plucked from physics and mathematics.

At one point he writes: "It is easy to gain the impression that there exists a huge landscape of mathematical structures, and that mathematicians explore this peculiar but inspiring territory, perhaps aided by the guiding hand of experience or the signpost of recent discoveries. Along the way these mathematicians come across new forms and theorems that are already there. The mathematician Rudy Rucker thinks of mathematical objects as occupying a sort of mental space--which he calls the 'Mindscape'--just as physical objects occupy a physical space. ... Occasionally different explorers will pass over the same terrain and report independently on their findings ... John Barrow also cites the phenomenon of independent discovery as evidence for 'some objective element' that is independent of the psyche of the

investigator."

Mathematical relations and theories are indeed "independent of the psyche of the investigator" but they are derived from the world of matter, not from the "Mindscape" of God, as Davies would have it.

The roots of Fermat's last theorem and of the abstract mathematics used by Wiles to solve it lie in the material world and in the efforts of mankind to raise the level of its productive forces.

It is no accident that the first solutions to the elementary equation $x^2 + y^2 = z^2$ were provided by the ancient Babylonians. The equation expresses the relationship between two sides of a right angled triangle to its longest side, known as the hypotenuse--it is commonly taught in high schools and known as Pythagoras' theorem.

The Babylonians, one of the first civilisations, required such tools of geometry to carry out the basic tasks of measuring fields and in the construction of buildings.

As Morris Kline comments in his book *Mathematical thought from ancient to modern times*: "Canals, dams and other irrigation projects required calculations. The use of bricks raised numerous numerical and geometric problems. Volumes of granaries and the areas of fields had to be determined. The close relationship between Babylonian mathematics and practical problems is typified by the following: A canal, whose cross-section was a trapezoid and whose dimensions were known, was to be dug. The amount of digging one man could do in a day was known, as was the sum of the number of men employed and the days they worked. The problem was to calculate the number of men and the number of days of work."

It is therefore no mystery that the germ of Fermat's theorem and therefore of Wiles' solution is contained on an old Babylonian baked clay tablet dating from about 1900-1600 BC known as Plimpton 322. It lists 15 Pythagorean triples, the first solutions to the equation, which prompted Fermat some 3,000 years later to make his famous, or perhaps infamous, note in the margin.

Frederick Engels explained in his famous book *Anti-Dühring*: "Pure mathematics deals with the space forms and quantity relations of the real world--that is, with material which is very real indeed. The fact that this material appears in an extremely abstract form can only superficially conceal its origin from the external world."

It is precisely because the origins of mathematics lie in the material world that its results can be applied to areas of human endeavour. Just as the non-Euclidean geometries developed in the nineteenth century were considered of interest only to a handful of mathematicians until Einstein's theories of relativity required their use, so it may be in the future that the discoveries of Wiles and his fellow number theorists will find practical application.



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